AD-A177 628 SIEVES AND SIGNAL EXTRACTION(U) WISCONSIN UNIV-MILWAUKEE DEPT OF MATHEMATICAL SCIENCES J H BEDER 29 OCT 86 AFOSR-TR-87-0213 AFOSR-84-0329 1/1

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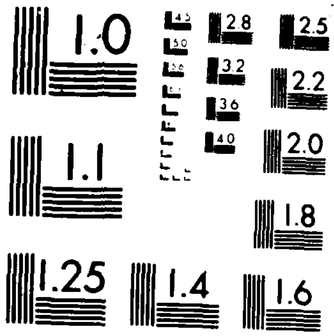

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AD-A177 628

REPORT DOCUMENTATION PAGE

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1a. REPORT SECURITY CLASSIFICATION: Unclassified	1b. RESTRICTIVE MARKINGS: None
2a. SECURITY CLASSIFICATION AUTHORITY: NA	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for Public Release; Distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE: NA	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 87-0213
6a. NAME OF PERFORMING ORGANIZATION: Dr. Jay H. Beder, Uni. of Wisconsin-Milwaukee; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: AFOSR/NM
6c. ADDRESS (City, State and ZIP Code): Department of Mathematical Sciences, P. O. Box 413, Milwaukee, WI 53201	7b. ADDRESS (City, State and ZIP Code): Bldg. 410, Bolling Air Force Base, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR; 8b. OFFICE SYMBOL (If applicable): NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR 84-0329
8c. ADDRESS (City, State and ZIP Code): Bldg. 410, Bolling AFB, DC	10. SOURCE OF FUNDING NOS.: PROGRAM ELEMENT NO. 6.1102F; PROJECT NO. 2304; TASK NO. A5; WORK UNIT NO.
11. TITLE (Include Security Classification): Sieves and Signal Extraction (Unclassified)	

12. PERSONAL AUTHOR(S): Beder, Jay H.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Annual Technical Report	FROM 9-30-85 TO 9-29-86	10-29-86	5

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Confounding, consistent estimation, nuclear operator, reproducing kernel Hilbert space, separability, sieve, stochastic signal, zero-one law.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Given a Gaussian signal with known mean and covariance, and given a second covariance kernel R, the PI has proved several results concerning the probability that the sample path of the signal will fall in the reproducing kernel Hilbert space with kernel R. This is applied to optimal extraction of an unobservable signal based on its conditional mean, and to the generalization of a zero-one law given by Kallianpur and by Driscoll.

For an observable Gaussian process with unknown mean and covariance, an extension of previous work shows that simultaneous consistent estimation of both the mean and the covariance is possible by the method of sieves. In both this and the signal extraction problem, no assumption is made about the nature of the "time" parameter of the process.

Some work on the axiomatic theory of confounding in experimental designs is also reported.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION: Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL: Maj. Brian Woodruff	22b. TELEPHONE NUMBER (Include Area Code): 202-767-5027; 22c. OFFICE SYMBOL: AFOSR/NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE

AFOSR-TR- 87-0213

Sieves and Signal Extraction (Unclassified)

1. Summary: See DD 1473, item 19.

2,3. Research objectives, status of work: Part of this project is to generalize the work of Driscoll [1973, 1975] on signal extraction. A brief description is given below; more detail may be found in the original project proposal.

To describe the current status of this work, let $\{S_t,\ t \in \Pi\}$ be a Gaussian process (signal) with mean 0 and known covariance K. Here Π is an arbitrary set. The sample path of the process is denoted $S_.$. Let R be any positive definite kernel with reproducing kernel Hilbert space (RKHS) $\mathcal{H}(R) = \mathcal{H}(R,\Pi)$, and assume that R dominates K ($K \ll R$), i.e. that $\mathcal{H}(K) \subset \mathcal{H}(R)$. Using the concept of n-domination ($K <^n< R$) (Fortet [1974]), we have:

Theorem A. If $K \ll R$ but not $K <^n< R$, then $P(S \in \mathcal{H}(R)) = 0$.

This generalizes a result of LePage [1973], for K = R.

The desired converse of Theorem A is: $K <^n< R \Rightarrow P(S \in \mathcal{H}(R)) = 1$, where the completion-measurability of the event $\{S \in \mathcal{H}(R)\}$ is part of the conclusion, as in Theorem A. Driscoll's proof of this result appears to be incomplete. Thus far we can show the following: Let $\mathcal{T}$ be the family of finite subsets of Π, and for each $T \in \mathcal{T}$ let

$$Z_T = \| S_. \|_T^2$$

(norm in $\mathcal{H}(R,T)$, where $S_.$ is restricted to T). Call $\{Z_T,\ T \in \mathcal{T}\}$ the derived process of S. Then by Fortet [1973] we see that the sets

$$\{S_{\cdot} \in \mathcal{H}(R)\} \text{ and } \{\sup_{T\in\mathcal{T}} Z_T\}$$

are equal - call the set A - and that on A we have $\|S_{\cdot}\|^2 = \sup Z_T$ (norm in $\mathcal{H}(R)$). The set A may not be measurable. Consider the weak separability assumption

$$(*)\quad \sup_{T\in\mathcal{T}} Z_T = \ell_{\cdot}\text{-th.}\sup_{T\in\mathcal{T}} Z_T \quad \text{a.s.}$$

(The lattice-theoretic supremum is defined in Tucker [1967].)

We have

<u>Theorem B.</u> If $K \overset{n}{<}\!< R$ and if (*) holds then $P(S_{\cdot}\in\mathcal{H}(R)) = 1$.

It is not yet clear just how weak or strong (*) is. It certainly holds in at least one of the cases Driscoll considers, so that we have his [1973] zero-one law. In fact, as long as (*) holds we have Kallianpur's zero-one law by letting K = R. However, our formulation also <u>characterizes</u> the "zero" and "one" parts of the law separately.

A second issue is optimal extraction of the signal S with regard to the norm of $\mathcal{H}(R)$. Let $\mathcal{A}_x$ be any σ-algebra of observable events such that $\hat{S}_t = E^{\mathcal{A}_x}(S_t)$ is Gaussian. The covariance C of $\hat{S}$ is dominated by K, and we can show:

<u>Theorem C</u>: If $K \overset{n}{<}\!< R$, then $C \overset{n}{<}\!< R$.

<u>Corollary</u>: If $K \overset{n}{<}\!< R$ and if the derived process of $\hat{S}$ satisfies (*), then $P(\hat{S}\in\mathcal{H}(R)) = 1$. It is not yet known whether it is sufficient to assume merely that S satisfies (*) in the Corollary. If both S and $\hat{S}$ satisfy (*) and if $K \overset{n}{<}\!< R$, we wish to show

$$(1)\quad E^{\mathcal{A}_X}\|S_\cdot - \hat{S}_\cdot\|^2 \le E^{\mathcal{A}_X}\|S_\cdot - h(\cdot)\|^2 \text{ for all } h \in \mathcal{H}(R),$$

and to evaluate the left hand side of (1).

Write $V_t = S_t - \hat{S}_t$, with covariance D, say (so $C + D = K$). Associate to K, C, and D the operators θ, θ_C, θ_D on $\mathcal{H}(R)$ (as in Fortet [1973]), all assumed nuclear, and satisfying $\theta = \theta_C + \theta_D$.

Theorem D. $E^{\mathcal{A}_X}\|S_\cdot - \hat{S}_\cdot\|^2 = \operatorname{tr}\theta_D$.

(The value $\operatorname{tr}\theta_C$ given in Driscoll [1975] is incorrect.)

Finally, to prove the inequality (1) we will need to show that a cross-product term $E^{\mathcal{A}_X}(S_\cdot - \hat{S}_\cdot, \hat{S}_\cdot - h)$ vanishes a.s. It is not yet clear how to do this.

We may mention one further result:

Theorem E. If $K \overset{n}{<} R << R_1$, then $K \overset{n}{<} R_1$. Thus there are many RKHS's $\mathcal{H}(R)$ whose norm could be used to define optimal signal extraction.

In Driscoll's original formulation, $\mathcal{A}_X$ was the σ-algebra of an observed signal X of form $X_t = S_t + N_t$, where N_t is independent Gaussian noise. The present formulation generalizes this, so that the unobservable signal S_t may be corrupted by noise in other ways.

The other part of this project involves sieve estimation for the mean and covariance of a Gaussian process. It has now been shown, in a joint paper with A. Antoniadis, that the two sieves can be combined to give simultaneous consistent estimation of the mean and the covariance functions. The original paper on mean estimation has been revised and will appear in Annals of Statistics. The paper on covariance estimation is being revised for that journal.

A joint paper with M. O'Laughlin gives an axiomatic theory of confounding which may be applicable to a variety of experimental designs including those in which the set of factor levels is arbitrary, possibly a continuum. In such cases one may view the "vector" of observations as a stochastic process.

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LePage, R. (1973). Subgroups of paths and reproducing kernels. Ann. Prob. 1: 345-347.

Tucker, H. (1967). A Graduate Course in Probability. Academic Press, N.Y.

4. Written Publications:

"A sieve estimator for the mean of a Gaussian process," Annals of Statistics (to appear in March 1987).

"A sieve estimator for the covariance of a Gaussian process," Annals of Statistics (in revision).

(with A. Antoniadis) Joint estimation of the mean and the covariance of a Banach valued Gaussian vector. Submitted to Journal of Applied Probability.

(with M. O'Laughlin) The cell-means interpretation of confounding. In manuscript.

5. Professional Personnel:

None besides the PI.

6. Interactions:

(a) Paper presented:

"Sieve estimation for Gaussian Processes," Colloquium, Dept of Mathematics, University of California, Irvine, June 10, 1986.

(b) Consultative and advisory functions: Joint research with A. Antoniadis on sieve estimation. Dept. of Mathematics, UC-Irvine, June 9-11, 1986.

7. New discoveries, inventions, patents: none.

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